Randomly Duty-cycled Wireless Sensor Networks: Dynamics of Coverage

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***Abstract*— This paper studies wireless sensor networks that operate in low duty cycles, measured by the percentage of time a sensor is on or active. The dynamic change in topology as a result of such duty-cycling has potentially disruptive effect on the performance of the network. We limit our attention to a class of surveillance and monitoring applications and random duty-cycling schemes, and analyze certain coverage property. Specifically, we consider *coverage intensity* defined as the probability distribution of durations within which a target or an event is uncovered/unmonitored. We derive this distribution using a semi-Markov model, constructed using the superposition of alternating renewal processes. We also present the asymptotic (as the number of sensors approaches infinity) distribution of the target uncovered duration when at least one sensor is required to cover the target, and provide an asymptotic lower bound when multiple sensors are required to cover the target. The analysis using the semi-Markov model serves as a tool with which we can find suitable random duty-cycling schemes satisfying a given performance requirement. Our numerical observations show that the stochastic variation of duty-cycling durations affects performance only when the number of sensors is small, whereas the stochastic mean of duty-cycling durations impacts performance in all cases studied. We also show that there is a close relationship between coverage intensity and the measure of *path availability*, defined as the probability distribution of durations within which a path (of a fixed number of nodes) remains available. Thus the results presented here are readily applicable to the study of path availability in a low duty-cycled sensor network.**

***Index Terms*— Energy conservation, microsensors, wireless sensor networks, coverage, connectivity, duty-cycling, alternating renewal process.**

I. INTRODUCTION

DRIVEN by advances in wireless communication and MEMS technology, a variety of applications using tiny, low cost, low power wireless sensors have emerged, ranging from environmental monitoring and global climate studies, to homeland security, industrial process control, and medical surveillance. In this paper we consider a class of surveillance and monitoring applications, whereby sensors deployed in a field or environment are used to monitor the presence/occurrence of some target/event of interest. Upon observation of such, a sensor generates a message and forwards it to a gateway (or control center), either directly or via multi-hopping, located somewhere in or near the network. As these sensors typically run on battery power, and the cost of replacing or recharging the sensors is usually high, it is critical to operate these sensors in a highly energy-efficient manner in order to ensure sufficient lifetime.

Many energy-saving approaches have been proposed in the literature. These approaches range from low-power device design, to energy-efficient networking, to advanced data compression and signal processing techniques. In this paper we will focus on one type of approach that centers on *duty-cycling* the sensors, i.e., turning off sensors periodically to conserve energy. By letting the sensors function at a low *duty cycle* – the fraction of time they are on, the sensors may last much longer. Such a system will be referred to as a low duty-cycled system. The price we pay for prolonged lifetime is potential performance degradation. Turning off the sensory device inevitably results in intermittent monitoring coverage, while turning off the wireless radio affects the connectivity of the network. Both affect the situational awareness and the responsiveness of the network. For instance, extra delay in packet forwarding may be encountered for lack of an active relaying sensor node.

One way to alleviate this negative effect on network performance is to deploy sensors in large quantities, i.e., redundancy in deployment[1], so that individual sensors may operate in lower duty cycles without affecting network performance. It is also possible for a sensor to be equipped with a dual-radio, of which one is used for data transmission and the other, a low power one, for paging. In this case a sensor can potentially "wake up" another sensor via the paging channel, see for example [1], [2]. While the use of a dual-radio may lead to simpler and more effective duty-cycling operations, it is more subject to jamming and more costly. For the rest of our discussion we will limit our attention to the single-radio scenario.

The central design problem in a low duty-cycled system is the determination of when to turn off a radio, or a sensory device, or both, and for how long. While solutions are in general application dependent, they can be loosely categorized into two main types: *random duty-cycling*, where sensors are turned on and off in a random fashion independent of each other, and *coordinated duty-cycling*, where sensors coordinate via communication and information exchange to collectively

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[1]The basic assumption here is that wireless sensors will become cheap enough for such redundant deployment to be feasible and effective. As the hardware technology improves, the cost of wireless sensors is expected to drop.

achieve an on/off (or wake/sleep) schedule followed by multiple sensors. There are also various combinations of these two methods. The advantage of the former is its simplicity while the latter can be potentially more efficient.

Our primary goal in this paper is to gain certain analytical understanding of the performance implications of duty-cycling for surveillance and monitoring applications. We will focus on random duty-cycling – which is more amenable to analysis and can also provide a performance lower bound on coordinated duty-cycling – and study performance measures related to network coverage and connectivity. Much of related prior work has studied the coverage and connectivity properties of static *snapshots* of the network, i.e., by examining the network at a particular time instant when some nodes are on and others are off as a result of duty-cycling; see for example [3], [4] (more is discussed in Section VII). This paper, on the other hand, focuses on the *dynamics* of coverage and connectivity properties over time, as nodes alternate between on and off modes.

Specifically, we are interested in the probability distribution of durations within which a *target* or an *event* is not covered/monitored, referred to as the *coverage intensity*. Unlike the commonly studied probability that a target is uncovered, coverage intensity reveals how likely a target is uncovered for a certain period of time. Thus it provides more information on the vulnerability of the network. We also show that there is an interesting relationship between coverage intensity and *path availability*, defined as the probability distribution of durations during which a path (consisting of a fixed number of nodes) remains available. Therefore results derived for coverage intensity readily applies to the study of path availability in a low duty-cycled sensor network.

Coverage intensity and path availability are fundamental to the understanding of the effect duty-cycling has on the coverage and connectivity properties of the sensor network. In this paper we study these measures via two different approaches. One is modeling based, by constructing a semi-Markov representation of the network using superposition of alternating renewal processes. The other is an asymptotic approach, by letting the number of sensors approach infinity. We present the asymptotic distribution of the target uncovered duration when at least one sensor is required to cover the target, and provide a tractable asymptotic lower bound when multiple sensors are required to cover the target. These two approaches complement each other – the former can be used to model a finite network but has a numerical result, while the latter provides good insight via closed form solutions but is only applicable to very dense networks. Furthermore, the analysis using the semi-Markov model serves as a tool to help us find suitable random duty-cycling schemes satisfying a given performance requirement. Our numerical observations show that the stochastic variation of on/off durations affects performance only when the number of sensors is small, whereas the stochastic mean of on/off durations affects performance in all cases we studied. This is further confirmed by our asymptotic results, which are only functions of the mean of these durations.

For simplicity of exposition, we will often use the term *duty-cycling* to refer to either turning on/off the radio transceivers or the sensory devices, or both. Unless otherwise specified, turning on/off the sensory devices is implied within the context of coverage, and turning on/off the radio is implied within the context of path availability. The rest of this paper is organized as follows. Section II gives the problem formulation and assumptions used throughout this paper. Section III derives coverage intensity under a discrete-time assumption and Section IV gives its asymptotics under a continuous-time assumption. Using the same approach, in Section V we discuss similar results on path availability; we also discuss the limitation of our approach and possible extension to random sensing and communication models. Section VI compares analytical results with that of simulation. We summarize related work in Section VII, and Section VIII concludes the paper.

II. Network Model and Assumptions

Sensors are assumed to be static once deployed. A sensor when active is able to monitor (we will also use the term *cover* interchangeably) a certain *event* or *target*. The coverage/sensing capability of a sensor is not necessarily associated with a geographic region, in that we are not assuming a fixed region within which event occurrences can be detected. Rather we simply assume that a sensor has the capability of detecting some event/target, leaving the location, range, or feature of such an event unspecified. In this sense the definition of an event or a target is rather broad; it does not have to be a point or of any particular shape. The key assumption is that the coverage capability is deterministic rather than probabilistic. That is, an event is either covered with probability 1 or probability 0. When we say that a target can be covered by a sensor, we mean that the sensor can detect/monitor the target with probability 1 when it is active/on. Similarly, when we consider path availability, we will assume that a sensor has a deterministic communication model such that it can directly communicate with another node with either probability 1 or 0. Sensors are assumed to follow an independent random sleep (or on/off) schedule, where each sensor selects on and off periods from certain probability distributions independent of other sensors. A target's *coverage degree* (CD) is defined as the number of sensors that can cover the target, and its *active coverage degree* (ACD) is defined as the number of active sensors among those that can cover the target.

We are primarily interested in the dynamics of the coverage property of the network as sensors are turned on and off, studied via the concept of *coverage intensity*. It is defined as the probability distribution of the time duration within which a target is uncovered. We will also be interested in *path availability*, defined as the probability distribution of the time duration within which a path is available. Under this definition a path is available only when all of its component links are available. Note that in general a path may be considered available as long as a packet can traverse from the source to the destination of the path, even if not all component links are available at the same time (i.e., some links may be up or down but a packet may still reach the destination after certain delay because the down links will eventually become active). Thus our definition of path availability is more restrictive.

The on/off schedule of an individual sensor can be modeled as an alternating Markov renewal process, having an *on* process and an *off* process. The collective effect of multiple sensors can be modeled as the superposition of multiple alternating Markov renewal processes. How to superpose these processes depends on the underlying objective of the study. For example, if we are interested in the coverage of an event with CD of n and with a coverage requirement of ACD being at least 1, then the superposed process is considered *on* if at least one of the component processes is *on*, and it is *off* if and only if all of the component processes are *off*. On the other hand, if we are interested in the availability of a path that consists of n sensors, then the superposed process is considered *on* if and only if all the component processes are on, and it is *off* if at least one of the processes is off. Thus there is an interesting *dual* relationship between coverage intensity and path availability under our model and definition. We will explore this further in subsequent sections.

III. Coverage Intensity

In this section we will model the on/off schedules of individual sensors as alternating Markov renewal processes (MRP), and examine the superposition of multiple such processes. While renewal theory is a well-established subject (e.g., see [5]), there are relatively fewer results on alternating renewal processes. In [6] the superposition of alternating renewal processes was studied with an application to statistical multiplexing of bursty traffic sources. In this section we employ the approach used in [6] to derive coverage intensity. We also present a simplified semi-Markov model with a linear state space, whereas the model based on [6] has an exponential state space.

A. Superposition of Markov Renewal Processes

We will assume discrete time, and thus the on and off periods are integer-valued and selected from certain probability mass functions (pmf) (having finite support) $f_i^{\text{on}}(k)$ and $f_i^{\text{off}}(k)$, $k = 1, 2, \cdots, K$, for some K, respectively. The same approach can be applied to continuous time in a similar way.

Consider n, $n \geq 2$, independent discrete-time MRPs. Each MRP has only 2 states, `off` (denoted as state 1) and `on` (denoted as state 2). The i-th MRP is characterized by a semi-Markov kernel $G_i(k) = [g_i(x, y, k)]$ defined over the set of states $\{1, 2\}$, where $g_i(x, y, k)$ is the probability that the i-th process goes from state x to state y in k slots where $x, y \in \{1, 2\}$. Thus we have

$$G_i(k) = \begin{bmatrix} g_i(1,1,k) & g_i(1,2,k) \\ g_i(2,1,k) & g_i(2,2,k) \end{bmatrix} = \begin{bmatrix} 0 & f_i^{\text{off}}(k) \\ f_i^{\text{on}}(k) & 0 \end{bmatrix}. \quad (1)$$

The superposition of n independent MRPs is modeled as a semi-Markov process. Note that this is an approximation since the future superposed state may depend not only on the present state and the time the superposed process has spent in the present state, but also on past states[2].

Define the state transition of the superposed process to occur at time instants when one or more of the component processes experience a state transition. A superposed state is given by the n-tuple

$$[(x_1, t_1), (x_2, t_2), \cdots, (x_n, t_n)], x_i \in \{1, 2\}, t_i \in \{0, 1\}, \quad (2)$$

where x_i is the state of the i-th process observed immediately *after* a transition occurs in the superposed process, and t_i indicates whether the i-th process has changed state when this transition occurs, with $t_i = 1$ iff process i has changed state and $t_i = 0$ otherwise. Denote by $\mathcal{S}$ the state space of the superposed process. The state space consists of all possible combinations of n pairs except when $t_i = 0$, $\forall i$, in which case no component process has a state transition and therefore the superposed process cannot have a state transition. The total number of states is thus $2^n(2^n - 1)$. Figure 1 illustrates an example of the superposition of two component MRPs ($n = 2$), and the corresponding state space $\mathcal{S}$.

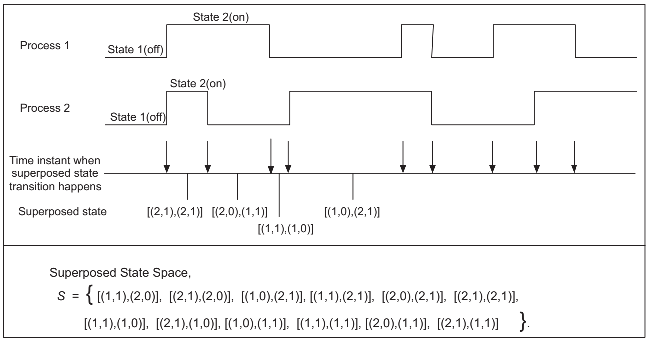

Fig. 1. A state transition example when there are $n = 2$ MRPs. State 1/2 is the off/on state.

A state $u \in \mathcal{S}$ is given by $u = [(x_1(u), t_1(u)), (x_2(u), t_2(u)), \cdots, (x_n(u), t_n(u))]$, where the i-th pair defines the state of the i-th component process when the superposed process transitions to state u. Consequently we will also refer to the i-th pair $(x_i(u), t_i(u))$ as the *state* of the i-th component process when the state of the superposed process is u. For example, if the superposed states are $u = [(2, 1), (2, 0)]$ and $v = [(1, 0), (1, 1)]$, then we have $(x_1(u) = 2, t_1(u) = 1)$ and $(x_1(v) = 1, t_1(v) = 0)$.

To obtain the distribution of the time the superposed process spends in state u before transitioning to state v, $u, v \in \mathcal{S}$, we begin with the following notations.

$g_i(x, y, k)$: the probability that the i-th component process stays in state x for k slots before transitioning to state y, where x and y denote the individual on/off states, $x, y \in \{1, 2\}$. This was previously given in Equation (1).

$\hat{g}_i(x, y, k)$: the probability mass function of the residual lifetime (or the forward recurrence time) when the renewal period has probability distribution $g_i(x, y, k)$, $x, y \in \{1, 2\}$. Using renewal theory, we have $\hat{g}_i(x, y, k) = \frac{1 - G_i(x,y,k-1)}{M}$, where $M = \sum_{k=0}^{\infty} k \cdot g_i(x, y, k)$ and $G_i(x, y, k) = \sum_{l=0}^{k} g_i(x, y, l)$.

$G_i(x, k)$: the cumulative probability that the the i-th component process' sojourn time in state x is up to k slots. It is given by $G_i(x, k) = \sum_{y \in \{1,2\}, y \neq x} \sum_{l=1}^{k} g_i(x, y, l) = \sum_{l=1}^{k} g_i(x, y, l),\ y \neq x,\ x, y \in \{1, 2\}$.

$\hat{G}_i(x, k)$: defined as $\sum_{y \in \{1,2\}, y \neq x} \sum_{l=1}^{k} \hat{g}_i(x, y, l) = \sum_{l=1}^{k} \hat{g}_i(x, y, l),\ y \neq x,\ x, y \in \{1, 2\}$.

[2]One exception is when the on and off distributions are memoryless, i.e., geometrically distributed, in which case the coverage intensity can be very easily obtained.

$q_n(u,v,k)$: the probability that the superposed process (of n MRPs) stays in superposed state $u = [(x_1(u), t_1(u)), \cdots, (x_n(u), t_n(u))]$ for k slots before transitioning to superposed state $v = [(x_1(v), t_1(v)), \cdots, (x_n(v), t_n(v))]$. The matrix $Q_n = [q_n(u,v,k)]$ denotes the semi-Markov kernel of the superposed process.

$P_n(u,v)$: the state transition probability of the superposed process when there are n MRPs. It is given by $P_n(u,v) = \sum_{k=1}^{\infty} q_n(u,v,k)$. The transition probability matrix is denoted by $\Lambda = [P_n(u,v)]$, $u, v \in \mathcal{S}$.

$P_n(v)$: the stationary distribution of the superposed process when there are n MRPs. The row vector of the stationary distribution is denoted by $V = [P_n(v)]$, $v \in \mathcal{S}$.

$\phi_i(u,v,k)$: the probability that the i-th component process stays in state $(x_i(u), t_i(u))$ for k slots before entering state $(x_i(v), t_i(v))$. Note that these are not necessarily distinct states for process i.

The stationary state distribution $V = [P_n(v)]$ can be obtained by solving $V\Lambda = V$, where Λ is determined by $P_n(u,v)$ which is in turn determined by $q_n(u,v,k)$. Therefore to find the stationary distribution, we need to find $q_n(u,v,k)$. Due to the independence of the component MRPs, we have

$$q_n(u,v,k) = \prod_{i=1}^{n} \phi_i(u,v,k) \ , \tag{3}$$

for all $u, v \in \mathcal{S}$, $u \neq v$, and $k \in Z^+$. Thus to compute $q_n(u,v,k)$, we only need to derive $\phi_i(u,v,k)$. Consider the i-th component process. Depending on the values of $t_i(u)$ and $t_i(v)$, the following four distinct cases are possible:

Case I: $t_i(u) = t_i(v) = 0$. In this case we have $x_i(u) = x_i(v)$, i.e., process i does not change state in either state u or v. The probability for such an event to occur is essentially the probability that the residual life-time of state $x_i(u)$ for process i is greater than k, and is given by $\phi_i(u,v,k) = 1 - \hat{G}_i(x_i(u), k)$.

Case II: $t_i(u) = 0$ and $t_i(v) = 1$. In this case process i changes state in v but not in u. Since process i has not changed state in u, the probability that this event occurs is the probability that the residual life-time of state $x_i(u)$ is equal to k. This is given by $\phi_i(u,v,k) = \hat{g}_i(x_i(u), x_i(v), k)$.

Case III: $t_i(u) = 1$ and $t_i(v) = 0$. In this case process i changes state in u but not in v. The probability that this event occurs is the probability that the sojourn time of state $x_i(u)$ is greater than k, and is given by $\phi_i(u,v,k) = 1 - G_i(x_i(u), k)$.

Case IV: $t_i(u) = t_i(v) = 1$. In this case process i changes state in both u and v. The probability that this event occurs is the probability that the sojourn time of state $x_i(u)$ before transitioning to state $x_i(v)$ is exactly k. This is given by $\phi_i(u,v,k) = g_i(x_i(u), x_i(v), k)$.

B. Coverage with Requirement ACD ≥ 1 — Single Sensor Coverage

Suppose that a target has CD of n, and its ACD has to be at least 1 in order for this target to be considered covered. The target is thus uncovered if all n sensors are off, corresponding to the superposed state $[(x_1, t_1), (x_2, t_2), \cdots, (x_n, t_n) : x_1 = x_2 = \cdots = x_n = 1]$. The set of all such states is denoted by $\mathcal{S}_{\text{uc}}$. The set of all states under which the target is covered is

$S_{UC} = \{ [(1,1),(1,0)],\ [(1,0),(1,1)],\ [(1,1),(1,1)] \}.$

$S_C = \{ [(1,1),(2,0)],\ [(2,1),(2,0)],\ [(1,0),(2,1)],\ [(1,1),(2,1)],\ [(2,0),(2,1)],\ [(2,1),(2,1)],\ [(2,1),(1,0)],\ [(2,0),(1,1)],\ [(2,1),(1,1)] \}.$

Fig. 2. An example of superposed uncovered state sets and superposed covered state sets when $n = 2$.

denoted by $\mathcal{S}_{\text{c}} = \mathcal{S} \setminus \mathcal{S}_{\text{uc}}$. Figure 2 gives an example of $\mathcal{S}_{\text{uc}}$ and $\mathcal{S}_{\text{c}}$ when $n = 2$.

It follows that, if a superposed process is currently in a state in $\mathcal{S}_{\text{uc}}$, then its next state is necessarily in $\mathcal{S}_{\text{c}}$. Since there are n sensors that can cover the target, the probability that the target is uncovered for k time slots, denoted by $P_n^{\text{uc}}(k)$, is given by:

$$P_n^{\text{uc}}(k) = \sum_{u \in \mathcal{S}_{\text{uc}}} \sum_{v \in \mathcal{S}_{\text{c}}} q_n(u,v,k) P_n(u), \quad \forall n \geq 2. \tag{4}$$

It can be easily obtained that $P_0^{\text{uc}}(k) = 1$ and $P_1^{\text{uc}}(k) = f_i^{\text{off}}(k) \cdot \frac{m_{\text{off}}}{m_{\text{off}} + m_{\text{on}}}$, where m_{off} is the mean of the off duration and m_{on} is the mean of the on duration.

C. Coverage with Requirement ACD $\geq m$ — Multiple Sensor Coverage

Consider the same scenario where a target has CD of n, but suppose that the coverage requirement is ACD $\geq m$, $2 \leq m \leq n$. Thus the target is uncovered if the number of off sensors is greater than $n - m$, corresponding to states of the superposed process $[(x_1, t_1), (x_2, t_2), \cdots, (x_n, t_n) : \sum_{i=1}^{n} I(x_i = 1) > n - m]$, where $I(\cdot)$ is the indicator function. Denote by $\mathcal{S}_{\text{uc}}^m$ the set of all such states, and by $\mathcal{S}_{\text{c}}^m = \mathcal{S} \setminus \mathcal{S}_{\text{uc}}^m$ the set of all states under which the target is covered. It might seem natural to compute the coverage intensity with the same equations as given in Section III-B by replacing $\mathcal{S}_{\text{uc}}$ and $\mathcal{S}_{\text{c}}$ with $\mathcal{S}_{\text{uc}}^m$ and $\mathcal{S}_{\text{c}}^m$, respectively. However, we will show that this leads to an under-estimate in Section V.B.

D. Applications

We will examine the accuracy of this modeling approach in Section VI. In this subsection we illustrate via examples how the model developed above can help us understand the performance degradation due to duty-cycling and compare different random on/off schedules with the same duty cycle. This comparison would not be possible if we only consider static snapshots of the network.

Suppose that a target has coverage requirement ACD ≥ 1. All on durations independently follow pmf $f^{\text{on}}(k)$, which is a uniform distribution over $[M_{\text{on}} - V_{\text{on}}, M_{\text{on}} + V_{\text{on}}]$ for some constants M_{on} and V_{on}. All off durations independently follow pmf $f^{\text{off}}(k)$, which is also uniform over $[M_{\text{off}} - V_{\text{off}}, M_{\text{off}} + V_{\text{off}}]$ for some constants M_{off} and V_{off}. Figure 3 presents numerical results of Equation (4), presented in the form of the tail distribution of the uncovered duration.

First, consider the scenario of $n = 2$. The left figure of $n = 2$ compares three random on/off schedules with the same average duty cycle but different distributions. In particular, random schedule $f^{\text{off}} = U[2-1, 2+1] / f^{\text{on}} = U[2-1, 2+1]$

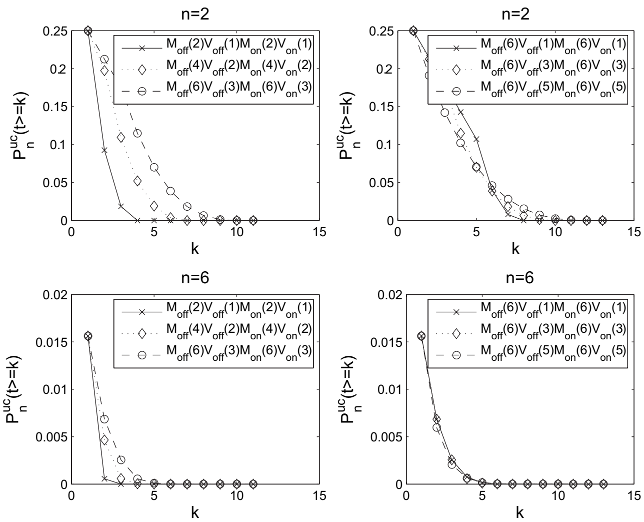

Fig. 3. Coverage intensity: numerical results obtained by equation (4).

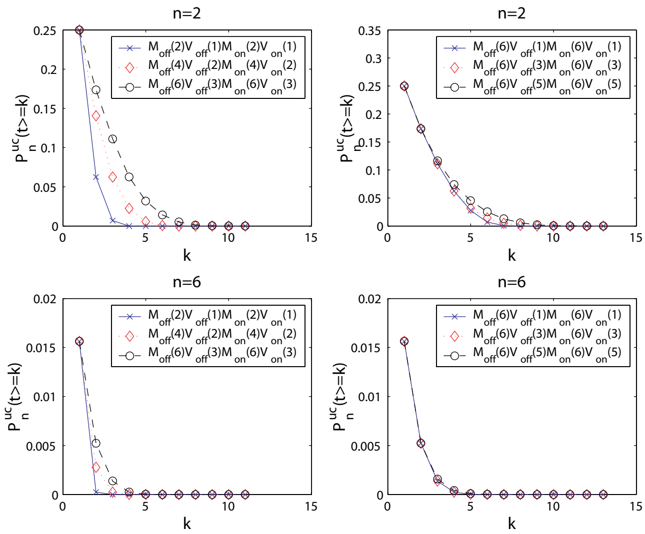

Fig. 4. Coverage intensity: numerical results obtained by equation (6).

may be preferred as it has the smallest uncovered probability for large (exceeding 4 in this example) periods of time. This is achieved by having sensors switch more frequently between on and off states. In practice, switching also consumes energy. Thus given the energy consumption model of particular sensor devices, the model presented here can be used to determine a desired switching frequency and the on/off distributions. The right figure of $n = 2$ compares three random on/off schedules with the same average on/off duration (thus the same average duty cycle and switching frequency) but different variances[3]. We see that the variance determines the shape of the tail distribution of the uncovered duration.

Next consider the scenario of $n = 6$ (which corresponds to higher node density compared to $n = 2$). From the right figure of $n = 6$ (and compared to the same on/off parameters when $n = 2$), the effect of different variance diminishes as n becomes larger. Thus when there is a large number of sensors that can cover a target, controlling the variance of on/off durations does not significantly affect the coverage intensity.

E. A Simpler Model with Reduced State Space

The semi-Markov model in Section III-A has an exponential state space, thus it does not scale well with the total number of nodes. Below we present a model with linear state space. We show that the simplified model provides satisfactory but coarser (less accurate) approximation compared to the previous model.

Consider n sensors, each of which can cover a target. Each node follows the same on/off distributions $f^{\text{on}}(k)$ and $f^{\text{off}}(k)$. Consider the following state space of the superposed process $\Omega = \{0, 1, \cdots, n\}$, where state i represents i off nodes, $i = 0, 1, \cdots, n$. This simplified semi-Markov model is also an approximation since the future superposed state may depend not only on the present state and the time the superposed process has spent in the present state, but also on past states.

Let $q_n(u, v, k)$ denote the probability that the superposed process stays in state $u \in \Omega$ for k slots before entering state $v \in \Omega$. For $u = n$, this probability can be approximated by the intersection of the following two events: that there are $n - v$ nodes with residual off time equal to k, and that there are v nodes with residual off time larger than k. Therefore,

$$q_n(n, v, k) = \binom{n}{n-v} [\hat{g}(1, 2, k)]^{n-v} \left[\sum_{l=k+1}^{\infty} \hat{g}(1, 2, l) \right]^v, \quad (5)$$

where $v = 0, \cdots, n - 1$ and $\hat{g}(x, y, k)$, $x, y \in \{1, 2\}$, was defined in Section III-A. Let $P_n(u)$ denote the stationary state distribution of superposed state $u \in \Omega$. Then the coverage intensity with requirement of ACD ≥ 1 is given by:

$$P_n^{\text{uc}}(k) = \sum_{v=0}^{n-1} q_n(n, v, k) P_n(n) = \sum_{v=0}^{n-1} \binom{n}{n-v} \times [\hat{g}(1, 2, k)]^{n-v} \left[\sum_{l=k+1}^{\infty} \hat{g}(1, 2, l) \right]^v \left(\frac{m_{\text{off}}}{m_{\text{off}} + m_{\text{on}}} \right)^n, \quad (6)$$

where m_{off} is the mean of the off duration and m_{on} is the mean of the on duration.

Figure 4 gives the numerical results of Equation (6), and they are very close to those in Figure 3. However, in the upper-right figure we see that the finer details of the effect of different variances seen in Figure 3 can no longer be observed. The advantage is that this simpler model provides a smaller state space ($n + 1$ states), compared to the original model ($2^n(2^n - 1)$ states). Furthermore, to calculate $P_n^{\text{uc}}(k)$, the computational complexity under this simpler model is $O(n!)$, while the original model has complexity $O(n \cdot 8^n)$.

IV. Asymptotic Coverage Intensity

In this section we examine coverage intensity in the asymptotic regime where the number of sensors goes to infinity. Different from the previous section, we will use a continuous-time assumption in this section for simplicity. As before, we will present the cases where a single sensor and multiple sensors are required for coverage, respectively.

The *asymptotic coverage intensity* examined in this section is defined as the probability distribution of the duration from

[3]The variances of the on/off durations are $\frac{1}{3}$, 3, and $\frac{25}{3}$, respectively.

an arbitrary time instant when a target is uncovered, to the first time instant that the target becomes covered, as CD (or n) goes to infinity. This definition is different from the one given in the previous section, which concerns the *entire* period of time during which the target is uncovered. In the asymptotic case we are only concerned with the *time to coverage* from an arbitrary time instant. This latter definition is more applicable in scenarios where we are interested in how long a target remains undetected from its first appearance.

We assume that each sensor independently alternates between on and off modes. All on and off durations are iid with probability density function (pdf) $f^{\text{on}}(x)$, $x > 0$, and $f^{\text{off}}(x)$, $x > 0$, respectively. The corresponding cumulative distribution functions (cdf) are denoted by $F^{\text{on}}(x)$ and $F^{\text{off}}(x)$, respectively. Assume that $F^{\text{on}}(x)$ and $F^{\text{off}}(x)$ are non-degenerate on $\mathbb{R}_+$ with $F^{\text{on}}(0) < 1$ and $F^{\text{off}}(0) < 1$. Assuming that these sensors start their operation at time $t \ll 0$, the on/off schedule of each sensor can be modeled as an equilibrium alternating renewal process.

A. Coverage with Requirement ACD ≥ 1 — Single Sensor Coverage

Suppose that a target has a CD of n and coverage requirement of ACD ≥ 1. Let $T_i(t)$ be the time that has elapsed from time t till the first moment that sensor i is on. Let E_1 be the event that the target is uncovered at time 0. Then $P[T_i(0) \leq x|E_1]$ is the distribution of the forward recurrence off time. Using standard results from renewal theory [5] and denoting this distribution by $H_i(x)$, we have:

$$H_i(x) = P[T_i(0) \leq x|E_1] = \frac{1}{m_{\text{off}}} \int_0^x (1 - F^{\text{off}}(y))dy \quad (7)$$

when $x > 0$, where m_{off} is the mean off duration, and $H_i(x) = 0$ when $x \leq 0$. Let X_i be a random variable distributed according to cdf $H_i(x)$. Let the random variable Z be such that $Z = \min_{i=1,\cdots,n} X_i$. Then Z denotes the duration from time 0 till the time the target is covered, given that it is not covered at time 0. In other words, this is the time during which the target is uncovered starting from some arbitrary time when it is not covered. The following theorem was proved in [7]. (More discussion on [7] is given in Section VII.)

Theorem 1: ([7])

$$\lim_{n\to\infty} P[Z \leq \frac{x}{n}] = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0 \end{cases}, \quad (8)$$

where $\lambda = \frac{1-F^{\text{off}}(0)}{m_{\text{off}}}$.

This is a fundamental result, which says that as n goes to infinity, the distribution of the random variable Z approaches that of an exponential random variable, regardless of the off duration distribution. Following the above theorem, the target's uncovered duration from time 0 given that the target is uncovered at time 0 has the probability distribution of $E_{\lambda n}$ as $n \to \infty$, where $E_{\lambda n}$ is an exponential random variable with rate λn and $\lambda = \frac{1-F^{\text{off}}(0)}{m_{\text{off}}}$.

Suppose that a target appears at time 0 (which is essentially an arbitrary time as the process started at $t \ll 0$). Then Z denotes the amount of time before it is detected, given it is not covered at time 0. Therefore the asymptotic coverage intensity defined here is the conditional forward recurrence uncovered duration.

B. Coverage with Requirement ACD $\geq m$ — Multiple Sensor Coverage

Consider the same scenario as in the previous subsection, but now suppose that a target has coverage requirement of ACD $\geq m \geq 2$, and it has CD $= n \gg m$. Denote by D_m the target's uncovered duration from time 0 given that the target is uncovered at time 0. We have the following result (proof is given in the appendix).

Theorem 2: If $n \gg m \geq 2$, then

$$\lim_{n\to\infty} P[D_m > \frac{x}{n}] \geq \sum_{k=0}^{m-1} \left(\frac{m_{\text{on}}}{m_{\text{on}} + m_{\text{off}}} \right)^k \cdot \frac{(\lambda x)^k}{k!} e^{-\lambda x}, \quad (9)$$

where $\lambda = \frac{1-F^{\text{off}}(0)}{m_{\text{off}}}$ and $x \geq 0$.

This result indicates that $P[D_m > \frac{x}{n}]$ is lower bounded by the sum of $\left(\frac{m_{\text{on}}}{m_{\text{on}}+m_{\text{off}}}\right)^k$ multiplying the probability of k Poisson arrivals with rate λn within time duration $\frac{x}{n}$. The Poisson-like lower bound can be roughly explained as follows[4]. Suppose that all n sensors are off at time 0. Denote the first time the i-th sensor is turned on by η_i. Without loss of generality, assume that $\eta_1 \leq \eta_2 \leq \cdots \leq \eta_n$. From Equation (11) in the appendix, the quantities $\eta_{i+1} - \eta_i$, $i = 1, \cdot\cdot, n-1$, have identical exponential distributions with rate λn. Thus each of such on events can be regarded as a Poisson arrival with rate λn. Therefore, $D_m > \frac{x}{n}$ is closely related to the event of k $(0 \leq k \leq m-1)$ such Poisson arrivals within $\frac{x}{n}$, which has probability $\frac{(\lambda x)^k}{k!} e^{-\lambda x}$.

V. Discussion

A. Application to Path Availability

The methods used in the previous sections can be applied to path availability in a straightforward way. Recall that path availability is defined as the distribution of the duration during which a path is available, and a path is available when all its component links are available. A path is assumed to consist of n sensors or $n-1$ links. Thus for the path to be available, all n sensors need to be on, corresponding to states of the superposed process with tuple $[(x_1, t_1), (x_2, t_2), \cdots, (x_n, t_n) : x_1 = x_2 = \cdots = x_n = 2]$. Denote by $\mathcal{S}_{\text{a}}$ the set of all such states, and by $\mathcal{S}_{\text{ua}}$ ($\mathcal{S}_{\text{ua}} = \mathcal{S} \setminus \mathcal{S}_{\text{a}}$) the set of all states under which the path is unavailable. Figure 5 gives an example of $\mathcal{S}_{\text{a}}$ and $\mathcal{S}_{\text{ua}}$ with $n = 2$. Figure 6 further illustrates the relationship among different subsets of $\mathcal{S}$. Note that, if the current state of the superposed process is in $\mathcal{S}_{\text{a}}$, then the next state must be in $\mathcal{S}_{\text{ua}}$. Thus the probability that a path with n sensors is available for exactly k time slots, using quantities derived earlier, is given by $P_n^{\text{aval}}(k) = \sum_{u\in\mathcal{S}_{\text{a}}} \sum_{v\in\mathcal{S}_{\text{ua}}} q_n(u, v, k)P_n(u)$, similar to Equation (4). The asymptotic results derived earlier can be

[4] The explanation here is not precise as the bound is not tight.

$S_a = \{ [(2,1),(2,0)],\ [(2,0),(2,1)],\ [(2,1),(2,1)] \}.$

$S_{ua} = \{ [(1,1),(2,0)],\ [(1,0),(2,1)],\ [(1,1),(2,1)],\ [(1,1),(1,0)],\ [(2,1),(1,0)],\ [(1,0),(1,1)],\ [(1,1),(1,1)],\ [(2,0),(1,1)],\ [(2,1),(1,1)] \}.$

Fig. 5. An example of $\mathcal{S}_{\rm a}$ and $\mathcal{S}_{\rm ua}$ as subsets of the state space $\mathcal{S}$ of the superposed process with $n = 2$.

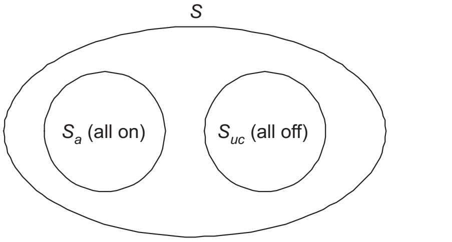

Fig. 6. An illustration of relationships among different subsets of the state space of the superposed process.

directly applied to obtain the asymptotic distribution of the conditional forward recurrence on time (i.e., the asymptotic path availability), by replacing $m_{\rm off}$ with $m_{\rm on}$ and $F^{\rm off}(0)$ with $F^{\rm on}(0)$.

B. Limitation and Extension

Consider the semi-Markov model and coverage intensity. As we noted earlier, if the current state of the superposed process is in $\mathcal{S}_{\rm uc}$, i.e., if the target is currently uncovered, then the next state is necessarily in $\mathcal{S}_{\rm c}$. Thus to go from uncovered to covered it takes only one state transition. The opposite, on the other hand, is not true. That is, if the target is currently covered, then the next state could be either in $\mathcal{S}_{\rm uc}$ or $\mathcal{S}_{\rm c}$. What this implies is that to go from covered to uncovered it can take multiple state transitions, and there are many different paths of state transitions leading to an uncovered state. Suppose we are interested in the distribution of time during which the target remains covered, and states u, v are such that $u \in \mathcal{S}_c$ and $v \in \mathcal{S}_{uc}$. Then $q_n(u, v, k)$ derived earlier only gives the probability that the process goes from u to v in k slots via *a single transition*. In order to obtain the above distribution, we need the probability that the process goes from u to v in k slots provided that v is the first uncovered state it enters. For $k = 1$, this probability is simply $q_n(u, v, 1)$. For $k = 2$, this probability is $q_n(u, v, 2) + \sum_{w \in \mathcal{S}_c, w \neq u} q_n(u, w, 1) q_n(w, v, 1)$. For $k = 3$, this probability is $q_n(u, v, 3) + \sum_{w \in \mathcal{S}_c, w \neq u} [q_n(u, w, 2) q_n(w, v, 1) + q_n(u, w, 1) q_n(w, v, 2)] + \sum_{w,z \in \mathcal{S}_c, w \neq z \neq u} q_n(u, w, 1) q_n(w, z, 1) q_n(z, v, 1)$. The complexity increases rapidly for even moderate values of k. The same problem exists for deriving the distributions of the target uncovered duration when A$CD \geq m > 1$ and of the *path unavailability* duration. The asymptotic version of the latter distributions are equally difficult to obtain, and are not immediately available from the results presented in Section IV.

The study here assumes that the sensing and communication models of a sensor are deterministic. In practice, due to the randomness in sensing, ambient noise and interference, probabilistic models more accurately describe a sensor's sensing and communication capability. For example, one can use received signal-to-noise ratio to determine whether a target is detected or whether a communication is successful, where the noise is modeled as a random variable. We will not further examine the details of these more general models (see for example [8], [9]), but simply note that these probabilistic models give the probability that a certain number of sensors can cover a given target. Once we have this probability, using the semi-Markov model to take randomness into account in deriving coverage intensity and path availability becomes straightforward.

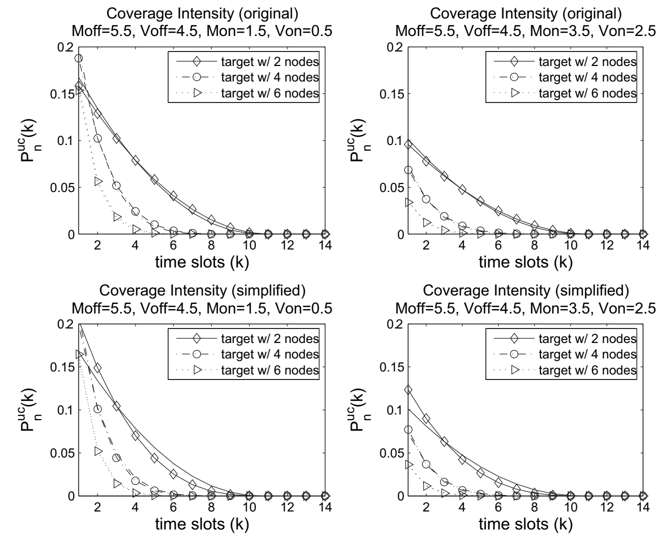

Fig. 7. Coverage intensity with requirement of ACD $\geq$ 1: results with/without markers are analytical/simulation results when $f^{\rm on}(k) \sim U[m_{\rm on} - v_{\rm on}, m_{\rm on} + v_{\rm on}]$ and $f^{\rm off}(k) \sim U[m_{\rm off} - v_{\rm off}, m_{\rm off} + v_{\rm off}]$. Top two: analytical results are from equation (4). Bottom two: analytical results are from equation (6).

VI. Numerical Experiments & Verification

In this section we present numerical and simulation results to evaluate the semi-Markov model as well as the asymptotic results. Whenever applicable, simulations are initialized using the equilibrium on/off distributions, which means that the first on/off period is distributed according to the forward recurrence time [5]. Most of the results shown here are for coverage intensity since path availability is essentially the same with exchanged on/off distributions.

Figure 7 compares the simulated coverage intensity with that obtained analytically using the semi-Markov model. $f^{\rm on}(k)$ is uniformly distributed over $[M_{\rm on} - V_{\rm on}, M_{\rm on} + V_{\rm on}]$, and $f^{\rm off}(k)$ is uniformly distributed over $[M_{\rm off} - V_{\rm off}, M_{\rm off} + V_{\rm off}]$. Coverage requirement is ACD ≥ 1. The simulation results are the averages of 10 runs[5]. In each run coverage intensity is calculated by $P_n^{\rm uc}(k) = [C_u(k) / \sum_m C_u(m)] \cdot (T_u/T)$, where T_u is the total time during which the target is uncovered, T is the total simulation time, and $C_u(k)$ is the total number of times that the target is uncovered for k slots. These are compared to those computed by Equation (4) in the top two figures of Figure 7 and by Equation (6) in the bottom two

[5]Technically a single run, given it's sufficiently long, would suffice, as the number of sensors is fixed and the randomness comes from the duty-cycling. In our case each run lasts for at least $7 \cdot 10^5$ on/off switches.

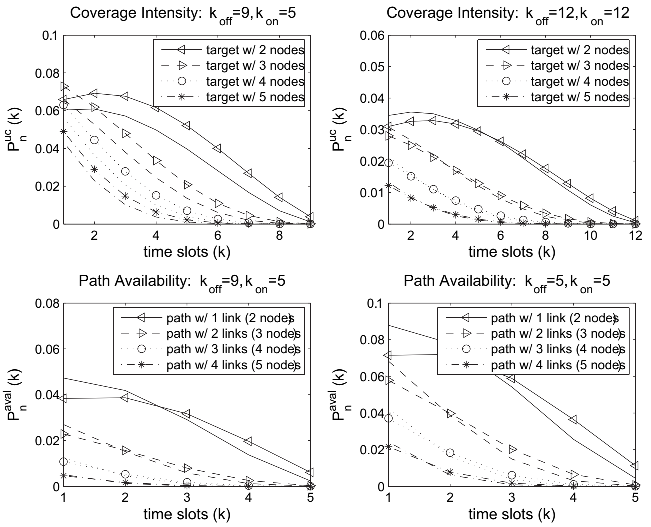

Fig. 8. Coverage intensity with requirement of ACD $\geq$ 1 and path availability: results with/without markers are analytical/simulation results when $f^{\mathrm{on}}(k) = k/\sum_{a=1}^{k_{\mathrm{on}}} a, 1 \leq k \leq k_{\mathrm{on}}$, and $f^{\mathrm{off}}(k) = k/\sum_{a=1}^{k_{\mathrm{off}}} a, 1 \leq k \leq k_{\mathrm{off}}$.

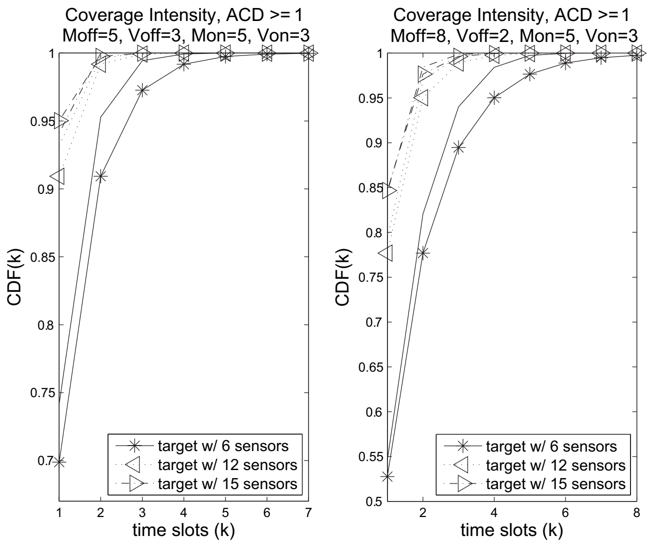

Fig. 9. Asymptotic coverage intensity with requirement of ACD $\geq$ 1: results with/without markers are analytical/simulation results when $f^{\mathrm{on}}(k) \sim U[m_{\mathrm{on}} - v_{\mathrm{on}}, m_{\mathrm{on}} + v_{\mathrm{on}}]$ and $f^{\mathrm{off}}(k) \sim U[m_{\mathrm{off}} - v_{\mathrm{off}}, m_{\mathrm{off}} + v_{\mathrm{off}}]$.

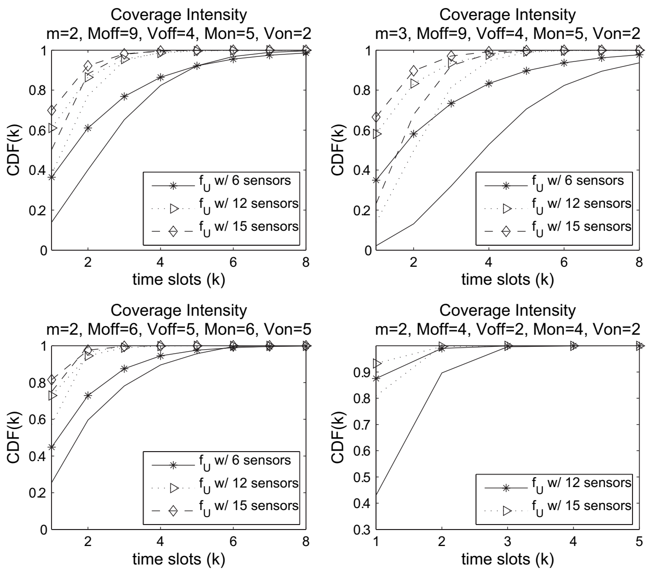

Fig. 10. Asymptotic coverage intensity with requirement of ACD $\geq m$: results with/without markers are analytical/simulation results when $f^{\mathrm{on}}(k) \sim U[m_{\mathrm{on}} - v_{\mathrm{on}}, m_{\mathrm{on}} + v_{\mathrm{on}}]$ and $f^{\mathrm{off}}(k) \sim U[m_{\mathrm{off}} - v_{\mathrm{off}}, m_{\mathrm{off}} + v_{\mathrm{off}}]$.

figures. We see that the analytical approximation performs better when the number of sensors (n) increases from 2 to 6. Overall the analysis matches well with simulation results for $f^{\mathrm{on}}(k)$ and $f^{\mathrm{off}}(k)$ considered here. As expected, the original model with exponential state space is slightly more accurate than the simplified model with linear state space. This observation remains true for all other experiments we conducted. For brevity, in the remainder of this section we will only present results using the original model.

Our second example uses the following distributions of on/off durations: $f^{\mathrm{on}}(k) = k/\sum_{a=1}^{k_{\mathrm{on}}} a, 1 \leq k \leq k_{\mathrm{on}}$, and $f^{\mathrm{off}}(k) = k/\sum_{a=1}^{k_{\mathrm{off}}} a, 1 \leq k \leq k_{\mathrm{off}}$. The top two figures of Figure 8 compare the simulated coverage intensity with that obtained analytically (via Equation (4)). We see that the accuracy improves as the number of sensors n increases. Furthermore, results for $k_{\mathrm{on/off}} = 5/9$ are less accurate than results of $k_{\mathrm{on/off}} = 12$ because the variance of the former on/off durations is smaller[6]. Smaller variance of on/off durations makes the future superposed state depend more on the past superposed states, causing the approximate semi-Markov model to be less accurate. The bottom two figures show the comparison for path availability, which leads to the same observation as coverage intensity. In general our model achieves fairly good approximation under many discrete-time, finite-support distributions of on/off durations.

Figure 9 compares the simulated and the analytical results on the asymptotic coverage intensity with requirement of ACD $\geq$ 1. It shows how the coverage intensity converges to the exponential distribution as the number of sensors increases. Simulation results are calculated based on 10^6 runs. Simulation starts from time -100. The conditional probability that the forward recurrence uncovered duration is k is calculated by $\frac{C_a(k)}{C_a}$, where C_a is the number of runs that the target is uncovered at time 0 and $C_a(k)$ is the number of runs that the forward recurrence uncovered duration from time 0 equals k. The analytical results are obtained using $P[Z \leq k] = 1 - e^{-\frac{1-F^{\mathrm{off}}(0)}{m_{\mathrm{off}}} kn}$, where n is the number of sensors. Note that here we are sampling the exponential distribution at discrete points.

Figure 10 shows how the asymptotic result of Theorem 2 provides a conservative bound on coverage intensity by multiple sensors. The conditional probability that the forward recurrence target uncovered duration is k is calculated by $\frac{C_u(k)}{C_u}$, where C_u is the number of runs that the target is uncovered (i.e., not covered by at least m sensors) at time 0 and $C_u(k)$ is the number of runs that the forward recurrence uncovered duration from time 0 equals k. The analytical lower bound on the tail distribution is shown in the graph as an upper

[6]The variance under $k_{\mathrm{on/off}} = 12$, $k_{\mathrm{on/off}} = 9$, and $k_{\mathrm{on/off}} = 5$ are 8.556, 4.8889, and 1.5556, respectively.

bound on the cdf. Using Theorem 2, the cdf upper bound is $f_U(k,n,m) = 1 - \sum_{0 \le i \le m-1} (\frac{m_{\rm on}}{m_{\rm on}+m_{\rm off}})^i \frac{(\lambda k n)^i}{i!} e^{-\lambda k n}$, where n is the number of sensors and m is the ACD requirement. As shown in the top two figures, the asymptotic bound of $m = 2$ is tighter than the bound of $m = 3$. As shown in the bottom two figures, the asymptotic bound on the left is tighter than the bound on the right. This is because, in deriving the bound, certain events were not considered, e.g., the one illustrated in Figure 11(b). This happens more often when m is larger, and when on/off durations are shorter. We can also see that (especially in the bottom two figures) the bound becomes tighter when the number of sensors (n) increases. Thus the general observation is that the asymptotic bound in Theorem 2 becomes tighter when the number of sensors increases, when the on/off durations are larger, and when the multiple sensor requirement (m) decreases.

VII. Related Work

The general problem of coverage and connectivity has been very extensively studied, within subjects like stochastic geometry and percolation theory. For example, [10] gave the bounds on the probability of coverage as functions of node density and the coverage radius, and it has been shown (e.g., in [11]) that there exists a critical node density for an unbounded connected component.

There have been numerous studies and results on network connectivity using graph theory. Various sufficient and necessary conditions were derived on asymptotic connectivity in a network as the number of nodes goes to infinity, see for example [12], [13], [14]. In [3], the coverage and connectivity of a network of unreliable sensors on a grid with a Boolean sensing and communication model were studied. All these studies examine static snapshots of the network. [15] studied path duration in a mobile ad hoc network in which the dynamics of a path is due to the movement of nodes. It found via simulation that the probability distribution of path duration can be well approximated by an exponential distribution under the mobility models considered. [7] studied analytically the path available duration problem by assuming that individual link available durations (due to mobility) are iid random variables. This result is applicable to our single sensor coverage problem (i.e., when the requirement is ACD ≥ 1), assuming that the duty-cycling processes of sensors are iid. Relevant result from [7] was used and cited earlier in Section IV. However, this result does not apply to multiple sensor coverage problem (i.e., when the requirement is ACD $\ge m \ge 2$). In this paper we provided a lower bound for coverage intensity in this scenario.

In an earlier, more preliminary effort [16], we have used the semi-Markov model presented in Section III-A to derive coverage intensity when only a single active sensor is required. In this study we further investigated coverage by multiple sensors, as well as a simpler semi-Markov model with a much reduced state space. In addition, asymptotic results were not available in [16].

There are also various algorithmic and heuristic studies on coverage, connectivity, and low duty-cycling. For example, the Set K-cover algorithms (see e.g., [17], [18]) aim at providing coverage of a field with only a subset of the nodes. By using one such subset (known as *covers*) at a time, the nodes are effectively duty cycled. Heuristic algorithms in providing coverage and connectivity while duty-cycling the sensors can be found in e.g., [16], [19], [20] and [21], respectively.

VIII. Conclusion

We studied properties of coverage over time as functions of individual sensor on/off schedules under a random duty-cycling assumption. Specifically, we derived the coverage intensity, defined as the probability distribution of the time duration in which a target is uncovered. We modeled the on/off schedules as semi-Markov processes, and obtained a mathematical model which allows us to calculate the coverage intensity numerically with very good approximation accuracy. We also presented a lower-complexity model with reduced state space. We then studied the asymptotic version of this measure as the number of sensors tends to infinity. We also showed that there is a close relationship between coverage intensity and the measure of path availability, defined as the probability distribution of durations in which a path (of a fixed number of nodes) remains available. Thus models/results obtained for coverage intensity readily applies to the study of path availability.

Appendix
Proof of Theorem 2

Let Y_i be a random variable with cdf $H_i(x)$, the conditional forward recurrence off time of the i-th sensor given in Equation (7). Since Y_i, $i = 1, 2, \cdots, n$, are iid, define $H(x) := H_1(x) = \cdots = H_n(x)$. From Equation (7) we have that $nH(\frac{x}{n}) = \frac{n}{m_{\rm off}} \int_0^{x/n} (1 - F^{\rm off}(t))dt$. Let $\tau = \frac{n}{x}t$ and using the bounded convergence theorem, we have the following

$$\lim_{n\to\infty} nH(\frac{x}{n}) = \lim_{n\to\infty} \frac{x\int_0^1 (1 - F^{\rm off}(\frac{x\tau}{n}))d\tau}{m_{\rm off}}$$
$$= \frac{x\int_0^1 \lim_{n\to\infty}(1 - F^{\rm off}(\frac{x\tau}{n}))d\tau}{m_{\rm off}} = \frac{1 - F^{\rm off}(0)}{m_{\rm off}} x, \quad (10)$$

Let $\lambda = \frac{1-F^{\rm off}(0)}{m_{\rm off}}$. Let W_m, $m = 1, 2, \cdots, n$, be a random variable defined as the m-th smallest of $\{Y_i\}_{i=1,\cdots,n}$. Then for $x > 0$ and $n \gg m \ge 2$, we have

$$\lim_{n\to\infty} P[W_m > \frac{x}{n}, W_i \le \frac{x}{n}, i < m] = \lim_{n\to\infty} \binom{n}{m-1}$$
$$\times [H(\frac{x}{n})]^{m-1} [1 - H(\frac{x}{n})]^{n-m+1} = \frac{(\lambda x)^{m-1}}{(m-1)!} e^{-\lambda x}, \quad (11)$$

where the second equality comes from approximating a binomial distribution with a Poisson distribution.

Suppose that a target has coverage requirement of ACD $\ge m$. Below we calculate the distribution of the conditional forward recurrence uncovered duration. Let this duration be denoted by D_m. Denote the probability density function of $H(x)$ by $h(x)$. Let $\pi_i^{\rm on}(t_2, t_1)$, $t_2 \ge t_1$, denote the probability that sensor i is on at time t_2 given that it entered an on duration at time t_1. Since on durations are iid, $\pi_i^{\rm on}(t_2, t_1) = \pi^{\rm on}(t_2, t_1)$ for all i. By the nature of alternating renewal processes, we have that $\pi^{\rm on}(t_2, t_1)$ only depends on $t_2 - t_1$. Subsequently we will use the simpler notation $\pi^{\rm on}(t)$, where $t = t_2 - t_1$.

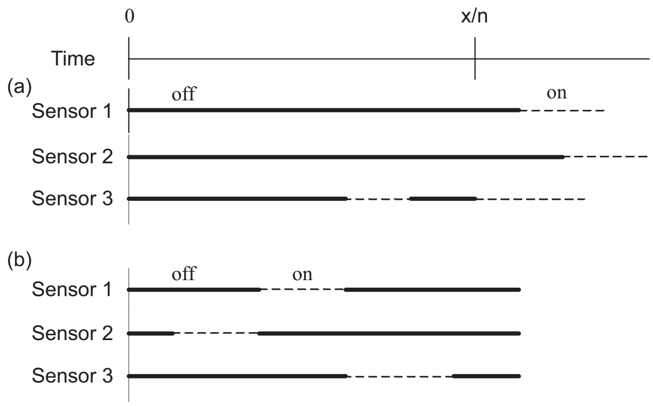

Fig. 11. Examples of $D_2 > \frac{x}{n}$ when there are 3 sensors and 2 on sensors are required to cover a target. (a)An example which is covered in the lower bound. (b)An example which is not covered in the lower bound.

From Section 7.3 of [5], $\pi^{\text{on}}(t)$ is a decreasing function of t, and $\pi^{\text{on}}(\infty) = \frac{m_{\text{on}}}{m_{\text{on}}+m_{\text{off}}}$. Furthermore, it is easy to see that $\pi^{\text{on}}(0) = 1$.

Define Z_{n-m} to be the random variable $\min_{i=1,\cdots,n-m} Y_i$, where Y_i's are iid random variables distributed according to $H(x)$. Thus Z_{n-m} has cdf $F_{Z_{n-m}}(x) = 1-[1-H(x)]^{n-m}$. Let $f_{Z_{n-m}}(x)$ be the pdf of Z_{n-m}. The event $\{D_m > \frac{x}{n}\}$ contains the union of disjoint events $E_k, k = 0, \cdots, m-1$, defined as follows: Among n sensors (off at time 0), (1) $n-k$ sensors have forward recurrence off time larger than $\frac{x}{n}$, and (2) the other k sensors have forward recurrence off time smaller than $\frac{x}{n}$ and are on at the time when the first sensor among the other $n-k$ sensors becomes on.

This union is only a subset of $\{D_m > \frac{x}{n}\}$, and this is shown in Figure 11. For $\{D_2 > \frac{x}{n}\}$, the event illustrated in Figure 11(a) is included in E_1 while the event illustrated in Figure 11(b) is not. As can be seen, there are many events similar to that in (b) and we have not found a tractable way to analyze these events. Below we proceed to derive $P[E_m]$.

$$\begin{aligned}
&\lim_{n\to\infty} P[E_m] \\
&\geq \lim_{n\to\infty} \binom{n}{m} \int_{\frac{x}{n}}^{\infty} \left[\int_0^{\frac{x}{n}} h(y_2)\pi^{\text{on}}(y_1-y_2)dy_2\right]^m f_{Z_{n-m}}(y_1)dy_1 \\
&\geq \lim_{n\to\infty} \binom{n}{m} \int_{\frac{x}{n}}^{\infty} \left[\int_0^{\frac{x}{n}} h(y_2)dy_2\right]^m [\pi^{\text{on}}(y_1)]^m f_{Z_{n-m}}(y_1)dy_1 \\
&= \lim_{n\to\infty} \left\{\binom{n}{m}[H(\frac{x}{n})]^m \int_{\frac{x}{n}}^{\infty} [\pi^{\text{on}}(y_1)]^m f_{Z_{n-m}}(y_1)dy_1\right\},
\end{aligned} \tag{12}$$

where the second inequality comes from the fact that $\pi^{\text{on}}(t)$ is a decreasing function of t. Integrating by parts, we obtain

$$\begin{aligned}
&\int_{\frac{x}{n}}^{\infty} [\pi^{\text{on}}(y_1)]^m f_{Z_{n-m}}(y_1)dy_1 = \left\{[\pi^{\text{on}}(y_1)]^m F_{Z_{n-m}}(y_1)\right\}_{\frac{x}{n}}^{\infty} \\
&- \int_{\frac{x}{n}}^{\infty} \phi(y_1)F_{Z_{n-m}}(y_1)dy_1,
\end{aligned} \tag{13}$$

where $\phi(y_1) = \frac{\partial[\pi^{\text{on}}(y_1)]^m}{\partial y_1}$. The first term on the right-hand-side is given by

$$\begin{aligned}
&\left\{[\pi^{\text{on}}(y_1)]^m F_{Z_{n-m}}(y_1)\right\}_{\frac{x}{n}}^{\infty} \\
&= \left\{[\pi^{\text{on}}(y_1)]^m \left[1-[1-H(y_1)]^{n-m}\right]\right\}_{\frac{x}{n}}^{\infty} \\
&= [\pi^{\text{on}}(\infty)]^m - [\pi^{\text{on}}(\frac{x}{n})]^m + [\pi^{\text{on}}(\frac{x}{n})]^m[1-H(\frac{x}{n})]^{n-m}.
\end{aligned} \tag{14}$$

For the second term on the right-hand-side of (13), we have $H(y_1) \geq H(\frac{x}{n})$, $\frac{x}{n} \leq y_1 \leq \infty$; thus $1-[1-H(y_1)]^{n-m} \geq 1-[1-H(\frac{x}{n})]^{n-m}$. This gives us $F_{Z_{n-m}}(y_1) \geq 1-[1-H(\frac{x}{n})]^{n-m}$. Noting that $\int_{\frac{x}{n}}^{\infty} \phi(y_1)dy_1$ is negative, we have

$$\begin{aligned}
&\int_{\frac{x}{n}}^{\infty} \phi(y_1)F_{Z_{n-m}}(y_1)dy_1 \\
&\leq \{1-[1-H(\frac{x}{n})]^{n-m}\} \int_{\frac{x}{n}}^{\infty} \phi(y_1)dy_1, \\
&= \{1-[1-H(\frac{x}{n})]^{n-m}\}\{[\pi^{\text{on}}(\infty)]^m - [\pi^{\text{on}}(\frac{x}{n})]^m\}.
\end{aligned} \tag{15}$$

Using Equations (14) and (15) in (13) and multiplying $\binom{n}{m}[H(\frac{x}{n})]^m$ on both sides, we have

$$\begin{aligned}
&\binom{n}{m}[H(\frac{x}{n})]^m \int_{\frac{x}{n}}^{\infty} [\pi^{\text{on}}(y_1)]^m f_{Z_{n-m}}(y_1)dy_1 \\
&\geq \binom{n}{m}[H(\frac{x}{n})]^m[\pi^{\text{on}}(\infty)]^m[1-H(\frac{x}{n})]^{n-m}.
\end{aligned} \tag{16}$$

Finally taking the limit of the above and using (11) and (12), we have

$$\lim_{n\to\infty} P[E_m] \geq \left(\frac{m_{\text{on}}}{m_{\text{on}}+m_{\text{off}}}\right)^m \cdot \frac{(\lambda x)^m}{m!}e^{-\lambda x}, \tag{17}$$

where $\lambda = \frac{1-F^{\text{off}}(0)}{m_{\text{off}}}$. Since $\{D_m > \frac{x}{n}\}$ contains the union of disjoint E_k, $k = 0, \cdots, m-1$,

$$\lim_{n\to\infty} P[D_m > \frac{x}{n}] \geq \sum_{k=0}^{m-1} \left(\frac{m_{\text{on}}}{m_{\text{on}}+m_{\text{off}}}\right)^k \cdot \frac{(\lambda x)^k}{k!}e^{-\lambda x}. \tag{18}$$

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